Problem of the Week
Problem C and Solution
Retiring and Hiring

Problem

A small technology company is beginning to expand. The company currently has 100 employees. At the end of each year for the next 4 years, 25 employees will retire and a new employee will be hired for each of the remaining employees.

After this four year cycle of retiring and hiring is complete, determine the average number of employees that the company grows by each year.

Solution

The following chart will look at the process of retiring and hiring over the four year period. The number remaining will be 25 less than the number at the start of the year. The number hired in a year will be the same as the number remaining. The number of employees at the end of the year will be twice as many as the number of employees remaining.

Year	# of Employees at Start of Year	# Retiring	# Remaining	# Hired	# of Employees at End of Year
1	100	25	75	75	150
2	150	25	125	125	250
3	250	25	225	225	450
4	450	25	425	425	850

After four years of retiring and hiring the company employs 850 people. The number of employees increased by $850 - 100 = 750$ people in four years. The average increase per year was $750 \div 4 = 187.5$ employees.

Another way to look at this problem would be to take the average number of people hired per year and subtract the average number of people retiring in a year. The average number hired per year is $(75 + 125 + 225 + 425) \div 4 = 850 \div 4 = 212.5$. The average retiring each year was 25. Therefore the average increase per year was $212.5 - 25 = 187.5$, as above.

Therefore the number of employees increased by approximately 188 per year.

Solution to Extension

Observe a couple of patterns in the column labelled "# of Employees at End of Year". First, each number ends in 50. The leading digits are 1, 2, 4, and 8. These digits are powers of 2. In year 2, the leading digit is 2^1. In year 3, the leading digit is 2^2. In year 4, the leading digit is 2^3. The exponent appears to be one less than the year number. So in year 10, a good prediction for the leading digits would be $2^9 = 512$. After 10 years of retiring and hiring, there would be 51 250 employees. (We can verify this prediction by continuing the table.)