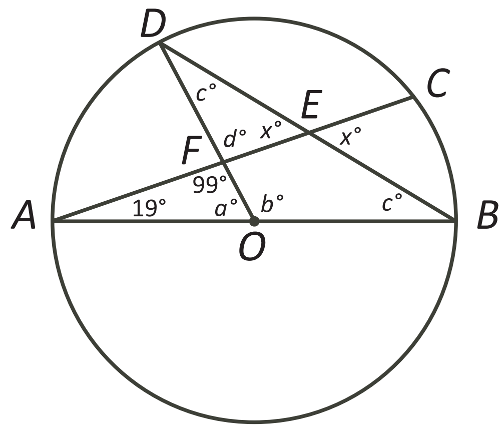
Problem of the Week
Problem C and Solutions
Circle-Go-Round

Problem

A circle with centre O is drawn around $\triangle OBD$ so that B and D lie on the circumference of the circle. BO is extended to A on the circle. Chord AC intersects OD and BD at F and E, respectively. If $\angle BAC = 19^\circ$ and $\angle OFA = 99^\circ$, determine the measure of $\angle BEC$. ($\angle BEC$ is marked x° on the diagram.)



Solution 1

In a triangle, the angles add to 180°. So in $\triangle OFA$,

$$\angle FOA = 180^\circ - 99^\circ - 19^\circ = 62^\circ = a.$$

BOA is a diameter and is therefore a straight line. Two angles on a straight line add to 180°, so

$$\angle BOD = 180^\circ - a^\circ = 180^\circ - 62^\circ = 118^\circ = b.$$

O is the centre of the circle with B and D on the circumference. Therefore, OD and OB are radii of the circle and $OD = OB$. It follows that $\triangle ODB$ is isosceles and $\angle ODB = \angle OBD = c$.

Then in $\triangle ODB$,

$$\begin{aligned} c^\circ + c^\circ + b^\circ &= 180^\circ \\ 2c + 118 &= 180 \\ 2c &= 62 \\ c &= 31 \end{aligned}$$

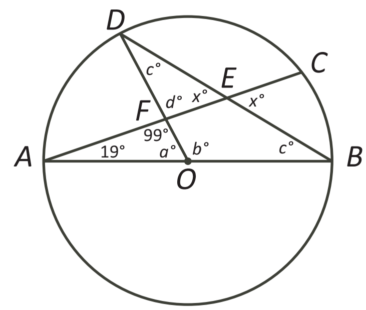
Opposite angles are equal so it follows that $\angle CEB = \angle DEF = x^\circ$ and $\angle DFE = \angle AFO = 99^\circ = d$.

In $\triangle FED$,

$$\begin{aligned} x^\circ + c^\circ + d^\circ &= 180^\circ \\ x + 31 + 99 &= 180 \\ x + 130 &= 180 \\ x &= 50 \end{aligned}$$

$\therefore \angle BEC = 50^\circ$.

Problem



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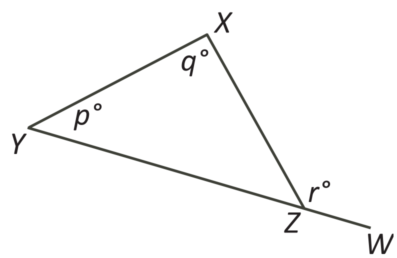
Solution 2

In a triangle, the angle formed at a vertex between the extension of a side and an adjacent side is called an exterior angle. In the following diagram, $\angle XZW$ is exterior to $\triangle XYZ$. The exterior angle theorem states: "the exterior angle of a triangle equals the sum of the two opposite interior angles." In the diagram, $r^\circ = p^\circ + q^\circ$. We will use this result and two of the pieces of information we found in Solution 1.



In a triangle, the angles add to 180°. So in $\triangle OFA$,

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O is the centre of the circle with B and D on the circumference. Therefore, OD and OB are radii of the circle and $OD = OB$. It follows that $\triangle ODB$ is isosceles and $\angle ODB = \angle OBD = c$.

$\angle FOA$ is exterior to $\triangle ODB$.

$$\begin{aligned} \therefore \angle FOA &= \angle ODB + \angle OBD \\ a^\circ &= c^\circ + c^\circ \\ 62 &= 2c \\ 31 &= c \end{aligned}$$

$\angle CEB$ is exterior to $\triangle EBA$.

$$\begin{aligned} \therefore \angle CEB &= \angle EAB + \angle EBA \\ x^\circ &= 19^\circ + c^\circ \\ x &= 19 + 31 \\ x &= 50 \end{aligned}$$

$\therefore \angle BEC = 50^\circ$.