



Problem of the Week

Problem C and Solution

A Powerful Problem For The New Year

Problem

5^3 is a *power* with *base* 5 and *exponent* 3.

5^3 means $5 \times 5 \times 5$ and equals 125 when expressed as an integer.

When 5^{2013} is expressed as an integer, what are the last three digits?

Solution

Let's start by examining the last three digits of various powers of 5.

$$5^1 = \quad \mathbf{005}$$

$$5^2 = \quad \mathbf{025}$$

$$5^3 = \quad \mathbf{125}$$

$$5^4 = \quad \mathbf{625}$$

$$5^5 = \quad \mathbf{3\ 125}$$

$$5^6 = \quad \mathbf{15\ 625}$$

$$5^7 = \quad \mathbf{78\ 125}$$

$$5^8 = \quad \mathbf{390\ 625}$$

Notice that there is a pattern for the last three digits after the first two powers of 5. For every odd integral exponent greater than 2, the last three digits are "125". For every even integral exponent greater than 2, the last three digits are "625". The pattern continues so 5^9 will end "125" since the exponent 9 is odd and 5^{10} will end "625" since the exponent 10 is even. This is easily verified since $5^9 = 1\ 953\ \mathbf{125}$ and $5^{10} = 9\ 765\ \mathbf{625}$.

For 5^{2013} , the exponent 2013 is greater than 2 and is an odd number.

\therefore the last three digits of 5^{2013} are 125.

