Problem of the Week
Problem C and Solution
This is Sum Problem

Problem

The number 90 can be expressed as the sum of 3 consecutive whole numbers. That is, $90 = 29 + 30 + 31$. The number 90 can also be written as the sum of 4 consecutive whole numbers. That is, $90 = 21 + 22 + 23 + 24$. Express the number 220 as the sum of 5 consecutive whole numbers and then as the sum of 8 consecutive whole numbers.

Solution

First, we want to express 220 as the sum of 5 consecutive whole numbers.

When 90 is expressed as the sum of 3 consecutive whole numbers, the average is $90 \div 3 = 30$. Since 30 is a whole number and there is an odd number of consecutive whole numbers in the sum, 29, 30 and 31 will produce the correct sum.

If we apply the same idea to 220, the average would be $220 \div 5 = 44$. Since 44 is a whole number and there is an odd number of consecutive whole numbers in the sum, 42, 43, 44, 45, and 46, should produce the correct sum. Checking, $42 + 43 + 44 + 45 + 46$, we obtain 220 as required.

Next, we want to express 220 as the sum of 8 consecutive whole numbers.

When 90 is expressed as the sum of 4 consecutive whole numbers, the average is $90 \div 4 = 22.5$. We need consecutive whole numbers such that two are below and two are above 22.5. This gives the numbers 21, 22, 23, and 24, as in the example. This would only work for an even number of consecutive whole numbers if the average is half way between two consecutive whole numbers.

Applying the same idea to 220, the average would be $220 \div 8 = 27.5$. This number is half way between 27 and 28. We would need four consecutive whole numbers below the average and four consecutive whole numbers above the average giving us the eight numbers 24, 25, 26, 27, 28, 29, 30, and 31. Checking, $24 + 25 + 26 + 27 + 28 + 29 + 30 + 31$, we obtain 220 as required.

As a concluding note, it is not possible to express 220 as the sum of four consecutive whole numbers. $220 \div 4 = 55$. The number 55 is the average of the four numbers but 55 is a whole number. We could try $54 + 55 + 56 + 57 = 222 \neq 220$ or $53 + 54 + 55 + 56 = 218 \neq 220$.

It is possible to express 220 as the sum of n consecutive whole numbers when n is odd provided that the average $220 \div n$ is a whole number.

It is possible to express 220 as the sum of n consecutive whole numbers when n is even provided that the average $220 \div n$ is half way between two consecutive whole numbers.